GCSE Maths – Geometry and Measures

Properties of Angles

Notes

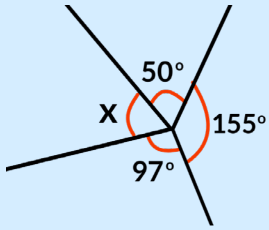
WORKSHEET

Properties of Angles

Angles at a point

The angles around a **point** add up to **360°**.

Example: Calculate the value of x in the diagram below



1. **Sum** the **angles** around the **point** and equate the sum to $360°$:

$$x + 50° + 155° + 97° = 360°$$

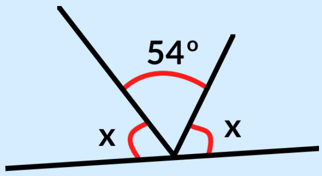
2. **Solve** the equation to find x:

$$x = 360° - 50° - 155° - 97°$$
$$\boldsymbol{x = 58°}$$

Angles on a straight line

The angles on a **straight line** add up to **180°**.

Example: Calculate the value of x in the diagram below



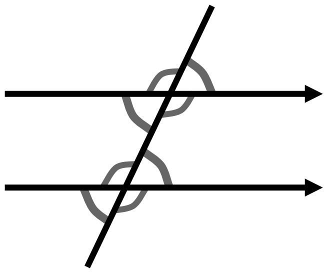
1. **Sum** the **angles** on the **line** and equate the sum to **180°**:

$$2x + 54° = 180°$$

2. **Solve** the equation to find x:

$$2x = 180° - 54°$$
$$2x = 126°$$
$$\boldsymbol{x = 63°}$$

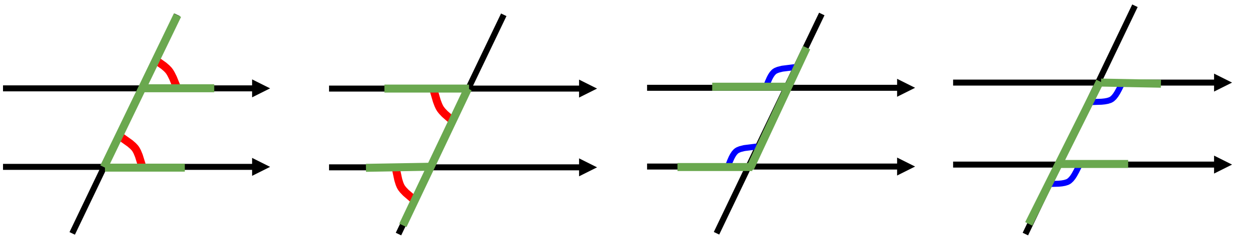
Alternate and corresponding angles



Parallel lines are lines which never meet and always lie the same distance apart from one another. When **parallel lines** are intersected by another line, it produces angles with important properties. The intersecting line is called a **transversal**.

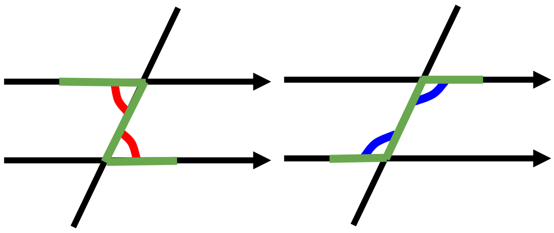
Corresponding angles

Corresponding angles are **equal**. They are on the **same side** of the transversal line and the lines on which the angles lie on form an **'F' shape**.



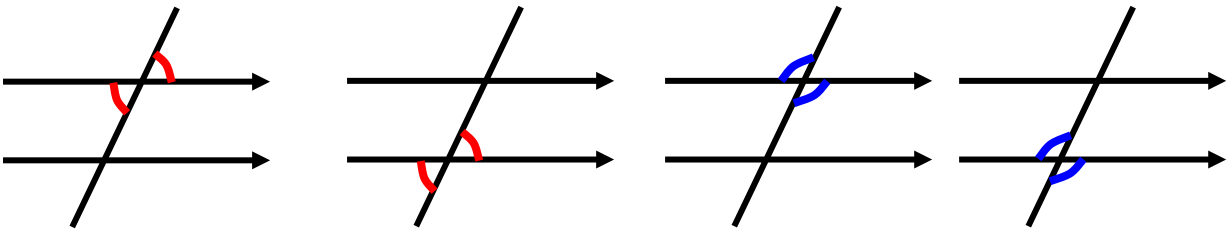
Alternate angles

Alternate angles are **equal**. They are on **opposite sides** of the transversal line and the lines on which they lie form a **'Z' shape**.



Vertically opposite angles

Vertically opposite angles are also **equal**.



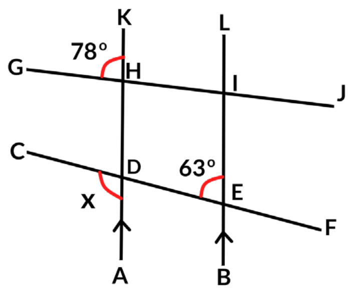
Example*:* Calculate the value of x in the diagram below

1. Find angle CDH.

   $Angle\ CDH = Angle\ DEI$ since they are corresponding angles. So $Angle\ CDH =$ $\mathbf{63°}$.

2. Find angle x by using the idea that angles on a straight line add up to 180°.

$$x + 63° = 180°$$
$$x = 180° - 63° = 117°$$



Note, that the properties only work on parallel lines. GJ and CF are not parallel, therefore, the properties cannot be used there.

Angles in regular polygons

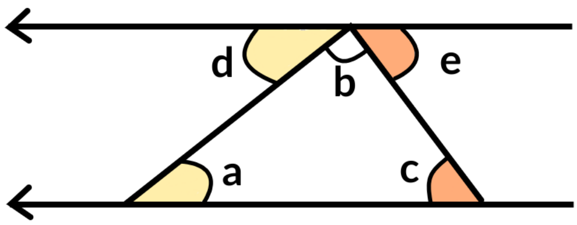
Regular polygons are polygons which have sides of equal length, and which have interior angles of equal value.

Interior angles

Interior angles are the angles found **inside** the polygon.

Interior angles in a triangle

The sum of the interior angles in a triangle add up to 180°. This can be proven using the idea of alternate angles:

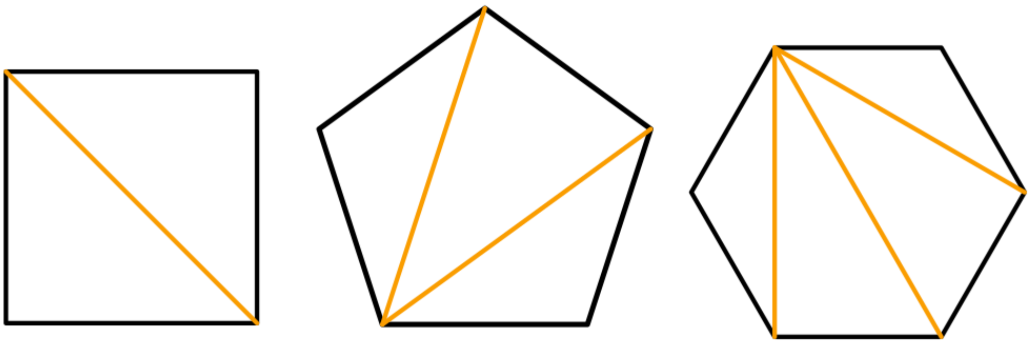


$Angle\ a = Angle\ d$ by alternate angles.
$Angle\ c = Angle\ e$ by alternate angles.

Since angles d, b and e all lie on a straight line, $d + b + e = 180°$. So, $a + b + c = 180°$.

Interior angles in any polygon

The sum of the interior angles in a polygon can be found by dividing the polygon into triangles.



- The **square** can be divided into **two triangles** so the interior angles of the square must add up to $2 \times 180° = 360°$.
- The **pentagon** can be divided into **three triangles** so the interior angles of the pentagon must add up to $3 \times 180° = 540°$.
- The **hexagon** can be divided into **four triangles** so the interior angles of the hexagon must add up to $4 \times 180° = 720°$.

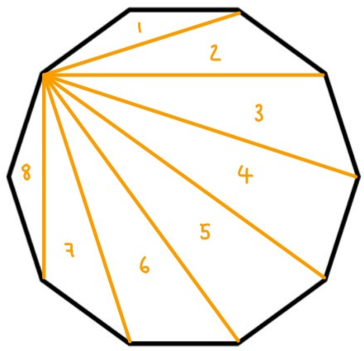
The formula for calculating the sum of interior angles in an n-sided polygon is

$$(n - 2) \times 180°.$$

Example*:* Calculate the sum of interior angles in a decagon

1. Count out how many triangles the polygon can be split into.

   As can be seen in the diagram, the decagon can be split into 8 triangles. The triangles are constructed by connecting one vertex to each of the other possible vertices.



2. Multiply the number of triangles by 180° to obtain the sum of the interior angles.

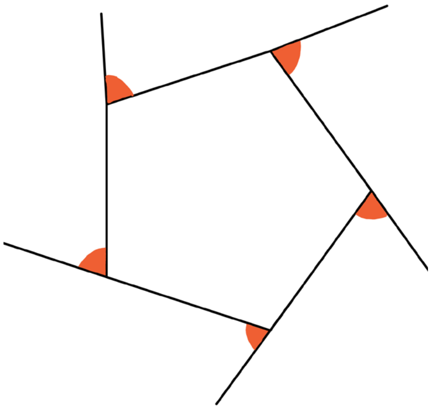
$$8 \times 180° = 1440°$$

Alternatively, you could use the formula $(n-2) \times 180°$ *with* $n = 10$ *since a decagon is a 10-sided polygon.*

Exterior angles

The angles formed outside the polygon when the sides of the polygon are extended are the **exterior angles.**

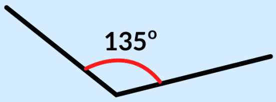
The **sum** of the exterior angles add up to **360°**.
This can be used to find the size of each exterior angle.



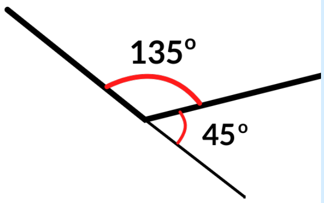
For an **n-sided** polygon:

$$Size\ of\ each\ exterior\ angle\ =\ 360° \div n$$

Example*: The diagram shows part of a regular polygon. Calculate how many sides the polygon has.*



1. Work out the value of the **exterior angle** by extending the side of the polygon and using the fact that **angles on a line** add up to 180°:



$$180° - 135° = 45°$$

2. Use the fact that the exterior angles **add up to 360°** to find the number of sides, n:

$$45^o \times n = 360^o$$

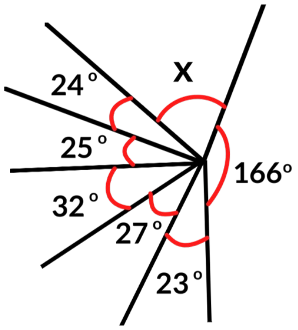
$$n = \frac{360}{45} = 8$$

*Therefore, the polygon has **8 sides** and is an **octagon**.*

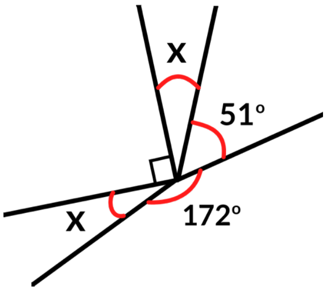
Properties of Angles – Practice Questions

1. Find angle x in each of the following diagrams:

a)

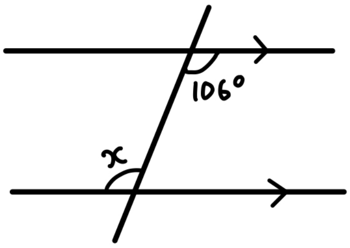


b)

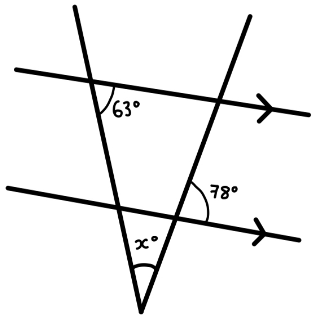


2. Find angle x in each of the following diagrams:

a)



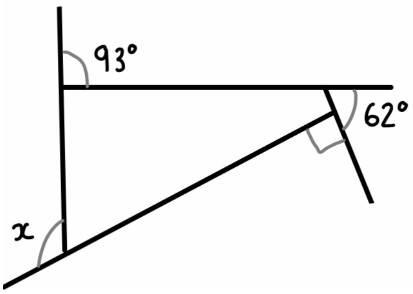
b)



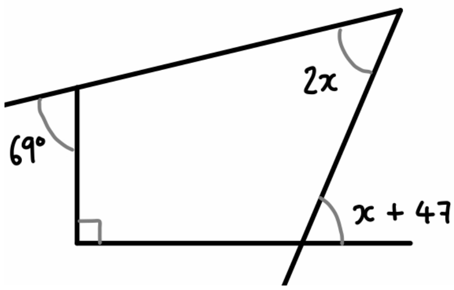
3. Calculate the sum of the interior angles in a polygon which has 23 sides.

4. In the following diagrams, find angle x:

a)



b)



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.